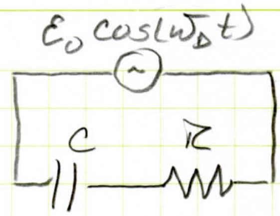
MT3 Pr 3-24

An electrical circuit has R & C in series with $\mathcal{E} = \mathcal{E}_0 \cos(\omega_D t)$. Find $I(t)$ and show it goes to zero as $\omega_D \to 0$



The D.E. is

$$R\dot{Q} + \frac{1}{C} Q = \mathcal{E}_0 \cos(\omega_D t)$$

Guess

$$Q(t) = A \sin(\omega_D t) + B\cos(\omega_D t)$$

$$\dot{Q}(t) = A\omega_D \cos(\omega_D t) - B\omega_D \sin(\omega_D t)$$

Stuff these into the DE

$$R\omega_D \left[A\cos(\omega_D t) - B\sin(\omega_D t)\right] + \\ + \frac{1}{C}\left[A\sin(\omega_D t) + B\cos(\omega_D t)\right] = \mathcal{E}_0 \cos(\omega_D t)$$

Equate coefficients of $\cos(\omega_D t)$ and $\sin(\omega_D t)$

$$R\omega_D A + \frac{1}{C} B = \mathcal{E}_0$$

$$-R\omega_D B + \frac{1}{C} A = 0 \Rightarrow A = CR\omega_D B$$

$$\Rightarrow R\omega_D (CR\omega_D B) + \frac{1}{C} B = \mathcal{E}_0$$

$$\left(CR^2\omega_D^2 + \frac{1}{C}\right) B = \mathcal{E}_0$$

$$\boxed{B = \frac{C\mathcal{E}_0}{C^2R^2\omega_D + 1}} \Rightarrow \boxed{A = \frac{C^2R\omega_D \mathcal{E}_0}{C^2R^2\omega_D + 1}}$$

Thus

$$Q(t) = \frac{C\mathcal{E}_0}{C^2R^2\omega_D + 1}\left[CR\omega_D \sin(\omega_D t) + \cos(\omega_D t)\right]$$

And

$$\boxed{I(t) = \frac{C\mathcal{E}_0\omega_D}{C^2R^2\omega_D + 1}\left[CR\omega_D \cos(\omega_D t) - \sin(\omega_D t)\right]}$$

And as $\omega_D \to 0$, the numerator in the first factor goes to zero and $I(t) \to 0$